Int. J. Heat Mass Transfer. Vol. 24, No. 10, pp. 1739–1742, 1981
Printed in Great Britain

0017 9310/81/101739–04 $02.00/0
Pergamon Press Ltd.

SHORTER COMMUNICATIONS

NATURAL CONVECTION IN THE ANNULI BETWEEN HORIZONTAL CONFOCAL ELLIPTIC CYLINDERS

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NOMENCLATURE

- a, focal length of ellipse (m);
- A, characteristic length (m);
- g, gravitational acceleration (m/s^2);
- G, gap ratio, (max. gap width + min. gap width)/(length of major axis of inner cylinder + that of minor axis);
- h, metric coefficient, h'/A;
- k_{eq}, local equivalent thermal conductivity, ratio of the heat transfer by convection to by conduction only;
- $\bar{k}_{eq}$, average equivalent thermal conductivity $[p^{-1}\int_0^p k_{eq_n}\,d\eta]$;
- Nu, Nusselt number $[h^{-1}(\partial T/\partial u)]$;
- p, perimeter of ellipse [m];
- Ra, Rayleigh number, $g\beta(T_i' - T_o' A^3/\alpha\nu)$;
- T, temperature $[(T' - T_o')/(T_i' - T_o')]$;
- u, radial independent variable;
- U, radial velocity, AU'/α;
- v, angular independent variable;
- V, angular velocity, AV'/α.

Greek symbols

- α, thermal diffusivity [m^2/s];
- β, thermal expansion coefficient [1/K];
- ε, eccentricity;
- ν, kinematic viscosity [m^2/s];
- φ, stream function, φ'/α;
- ω, vorticity.

Subscripts

- i, refers to inner cylinder;
- D, major axis length of inner cylinder used as length scale;
- L, maximum gap width in an annulus used as length scale;
- o, refers to outer cylinder;

Superscript

- pi, perimeter of inner cylinder used as length scale;
- $'$, denotes real variable.

INTRODUCTION

IN RECENT years, natural convection in an enclosed space has received increasing attention both theoretically and experimentally. However the behaviour of natural convection in a horizontal annulus is not well understood, except between circular cylinders.

This work is primarily concerned with natural convection in a horizontal annulus between confocal elliptical cylinders, which is the typical example of variable curvature, and it is possible to adopt the orthogonal elliptic cylindrical coordinate system. Since the analysis of elliptical cross-section has a generality which can include the case of circular cross-section as a particular case, the analysis has a wider application than the previous one. The annular geometry between elliptical cylinders can arise in a wall section of: (1) a double insulated receiver containing a cryogenic fluid or a dangerous substance which needs to be located in a limited space, (2) a double insulated vessel for above substance whose centre of weight needs to be lowered or whose height is limited for transportable trailers and railway cars.

T. H. Kuehn [1] extended the information about velocity and temperature distribution and local heat transfer coefficient with an excellent review of the annuli between horizontal concentric circular cylinders. In addition, he extended this study to a single horizontal cylinder and then compared his results with existing boundary layer solutions. The only study of elliptic cross-section in natural convection heat transfer was about a single horizontal elliptical cylinder by Lin and Chao [2], Raithby and Hollands [3, 4].

In this work using a governing equation which contains another non-linear parameter, eccentricity, in addition to the equation of a horizontal annulus with circular cross-section, the natural convective heat transfer between confocal elliptical cylinders has been studied numerically. In addition, using air as a test fluid, an experiment was performed by the Mach–Zehnder interferometer and smoke test method in order to verify the numerical study.

ANALYSIS

We consider a steady laminar 2-D natural convection with uniform temperature at each elliptical cylinder. According to the posture of annulus we quote "blunt" and "slender" configuration from Lin and Chao [2]. The non-dimensional governing equations in blunt configuration are:

$$\omega = -\nabla^2\varphi, \tag{1}$$

$$\nabla^2\omega = \frac{1}{h}Ra\left[\xi\left(\frac{T\sin\theta\sin 2v}{2} - \frac{T\cos\theta\sinh 2u}{2}\right) + \frac{\partial T}{\partial v}\sin\theta - \frac{\partial T}{\partial u}\cos\theta\right] + \frac{1}{h}\frac{1}{Pr}\left(U\frac{\partial\omega}{\partial u} + V\frac{\partial\omega}{\partial v}\right). \tag{2}$$

$$\nabla^2 T = \frac{1}{h}\left(U\frac{\partial T}{\partial u} + V\frac{\partial T}{\partial v}\right), \tag{3}$$

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where

$$U = h_0^{-1}\,\partial\varphi/\partial v, \quad V = -h_0^{-1}\,\partial\varphi/\partial u, \tag{4}$$

$$\xi = \frac{1}{\sinh^2 u + \sin\theta} - \frac{1}{\sinh^2 u \cos^2 v + \cosh^2 u \sin^2 v}, \tag{5}$$

$$\theta = \tan^{-1}\frac{\cosh u \sin v}{\sinh u \cos v}. \tag{6}$$

In slender configuration, equation (7) must be used instead of equation (2).

$$\nabla^2\omega = -\frac{1}{h}Ra\left[\xi\left(\frac{T\cos\theta\sin 2v}{2} - \frac{T\sin\theta\sinh 2u}{2}\right) + \frac{\partial T}{\partial v}\cos\theta + \frac{\partial T}{\partial u}\sin\theta\right] + \frac{1}{h}\frac{1}{Pr}\left(U\frac{\partial\omega}{\partial u} + V\frac{\partial\omega}{\partial v}\right). \tag{7}$$

The boundary conditions are;

$$\left.\begin{array}{l} v = \pm\pi/2 \quad \text{(for blunt configuration)} \\ v = 0,\ \pi \quad \text{(for slender configuration)} \end{array}\right\};$$

$$\varphi = V = \omega = \frac{\partial T}{\partial v} = \frac{\partial U}{\partial v} = 0, \tag{8}$$

$$u = u_i;\ \varphi = U = V = \partial^2\varphi/\partial v^2 = 0,\ T = 1, \tag{9}$$

$$u = u_o;\ \varphi = U = V = \partial^2\varphi/\partial v^2 = 0,\ T = 0. \tag{10}$$

EXPERIMENTAL APPARATUS

The apparatus consists of two confocal elliptical cylinders enclosed in a cooling water jacket. A single outer elliptical cylinder with $\varepsilon_o = 0.400$ and $a = 20$ mm was used in conjunction with three inner elliptical cylinders of $\varepsilon_i = 0.688$, 0.574, 0.474. All cylinders were 250 mm long and 3 mm thick copper. Using a Mach–Zehnder interferometer whose mirror-diameter is 200 mm, the isotherms were taken. The streamlines were photographed by a simple smoke test method.

RESULTS AND DISCUSSION

The generality of this analysis was checked by forming an almost-perfect concentric circular cross-section for which the length of the minor axis was above 99.5% of that of the major axis for the inner cylinder. Our result was compared with Kuehn and Goldstein [6] when $G = 0.8$, $Ra_L = 5 \times 10^4$ and $Pr = 0.7$. For the greater part of the annulus, both results agreed with each other in respect of average heat transfer rate, temperature distribution and velocity fields of flow. But the position of the maximum heat transfer coefficient of the inner cylinder was about $\pi/4$ in angular direction above the lower limit of our result.

Figure 1 shows the influence of Ra_L on the average equivalent thermal conductivity at $a = 2.11$ mm, $\varepsilon_i = 0.600$, $\varepsilon_o = 0.475$ and $Pr = 0.8$. The rate of heat transfer is greater in slender configurations than blunt ones in the same conditions. It is believed that these phenomena arise because the magnitude of flow velocity in the middle portion of the annulus has been influenced by curvature at that portion.

When the outer cylinder is much larger than the inner cylinder, the natural convection from the inner cylinder is similar to that from the single elliptical cylinder. We chose an annulus whose ratio of the average axis length of outer cylinder to inner cylinder was as much as 20.08. Ra_D of the annulus was chosen as 100 to provide a relatively thick boundary layer at the inner cylinder. The resulting local heat transfer rate is shown in Fig. 2 compared with the results using boundary layer equations. The qualitative trends are similar, but the magnitude of our result is about twice that of [6]. This indicates a lower limit range of Rayleigh number for which the application of the boundary layer equation to the single elliptic cylinder is not suitable.

Figure 3 shows the influence of eccentricity of inner

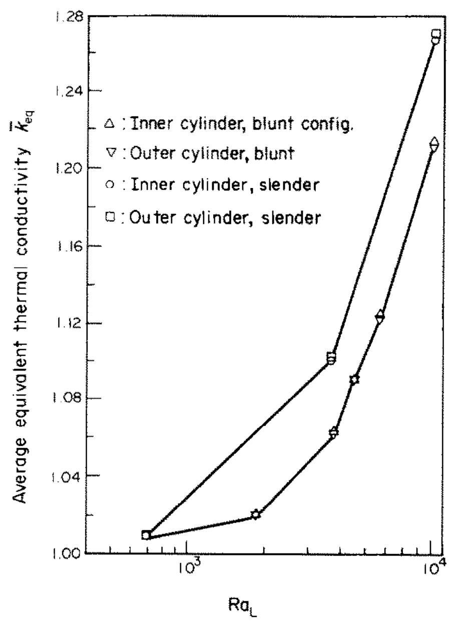

FIG. 1. Influence of Rayleigh number on average equivalent thermal conductivity.

cylinder on average equivalent conductivity at blunt configuration, when the ratio of the average axis length was about 6. For this purpose, we have constructed 9 annuli so that the area of heat transfer between outer and inner cylinders are the same by fixing the cross-sectional area of annuli as 61.1 cm^2. In the case of $\varepsilon_i = 0.9$, the average equivalent thermal conductivity is 9% lower than that of $\varepsilon_i = 0.1$. This is presumably caused by the fact that the natural convective flow is obstructed by the broadening area of horizontal projection of the inner cylinder with increasing eccentricity.

Among the experiments performed, one result with $G = 0.456$ was chosen for comparison with numerical results. Figures 4 and 5 show the isotherms and streamlines respectively. Fairly good agreements were obtained.

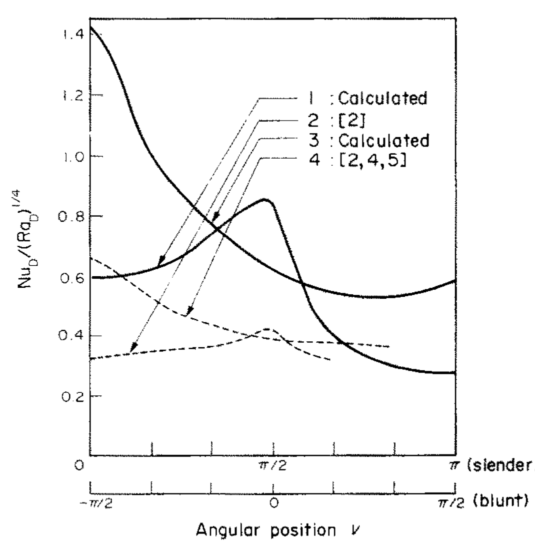

FIG. 2. Comparison of heat transfer coefficient at $Ra_D = 100$ with boundary layer solutions.

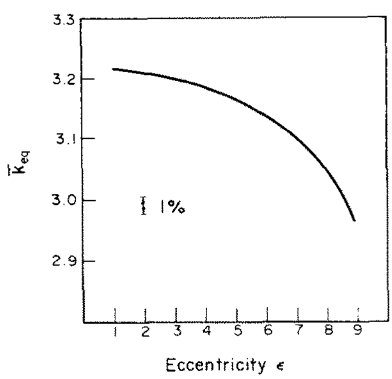

FIG. 3. Influence of eccentricity on the average equivalent thermal conductivity for Ra_{pi} = 105, Pr = 0.7.

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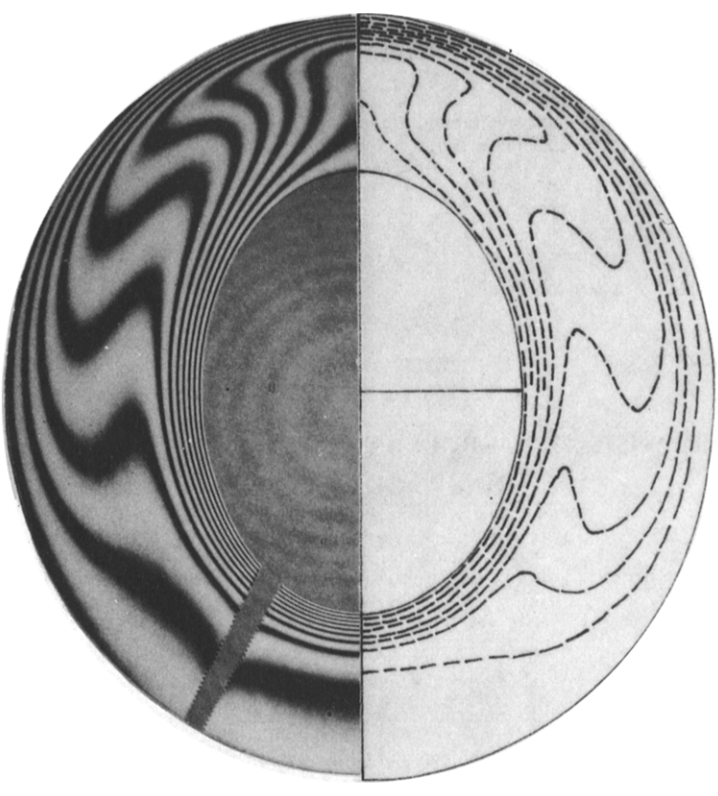

FIG. 4. Comparison between experimental and numerical temperature distribution at slender configuration for $Ra_L = 3.795 \times 10^4$, $Pr = 0.707$, $a = 20$ mm, $\varepsilon_i = 0.688$, $\varepsilon_o = 0.400$.

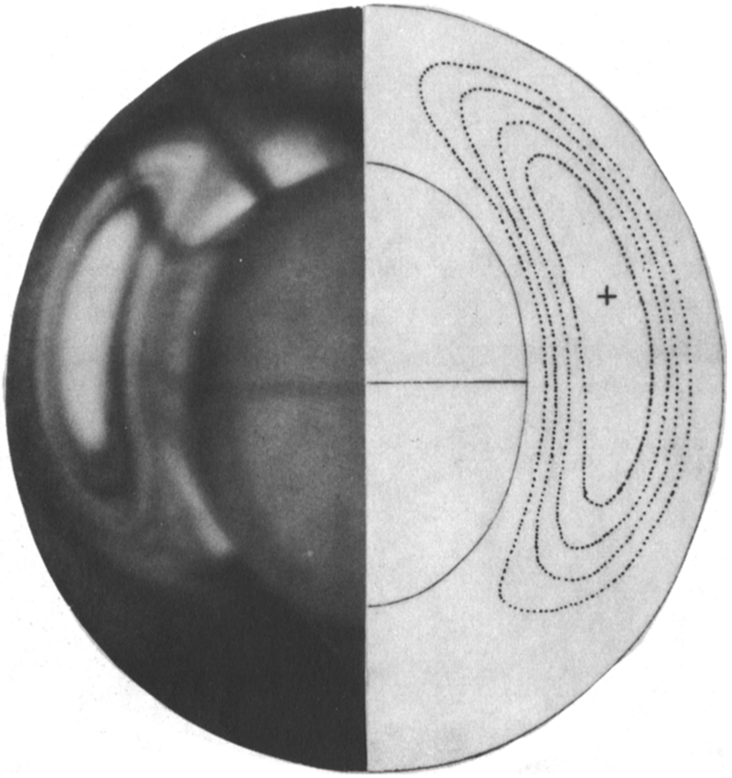

FIG. 5. Comparison between experimental and numerical streamlines at slender configuration for $Ra_L = 3.795 \times 10^4$, $Pr = 0.707$, $a = 20$ mm, $\varepsilon_i = 0.688$, $\varepsilon_o = 0.400$.

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Printed in Great Britain

0017-9310/81/101742-04 $02.00/0

TRANSIENT TURBULENT THERMAL CONVECTION IN A POOL OF WATER

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(*Received* 14 *October* 1980 *and in revised form* 27 *February* 1981)

NOMENCLATURE

C_j, constants, $1 \leqslant j \leqslant 5$;
c_p, specific heat at constant pressure;
D, dissipation-rate of turbulent kinetic energy;
g, gravitational acceleration;
k, kinetic energy;
K, thermal conductivity;
l, mixing length;
Nu, Nusselt number;
P, production-rate of turbulent kinetic energy;
Pr, Prandtl number;
q, heat flux;